An Exhaustive Theoretical and Phenomenological Analysis of Geometrically-Induced Mass Variation (GIMV)

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Abstract

This analysis treats the Geometrically-Induced Mass Variation (GIMV) framework [1] as a serious, testable, and internally consistent theoretical proposal. The implications of its central premise—that nucleon mass couples directly to the gravitational tidal invariant $\mathcal{K}$—are profound, touching every major domain of modern physics. We find the theory is a viable "strong-field-only" phenomenon, shielded by the $\sim$ 34 order-of-magnitude difference in the Kretschmann scalar $\mathcal{K}$ between terrestrial environments ($\mathcal{K} \sim 10^{-12}$ s^{-4}) and the surface of a neutron star ($\mathcal{K} \sim 10^{22}$ s^{-4}) [1]. This "viability gulf" allows the GIMV coupling constant ξ to be negligible for Equivalence Principle tests while remaining dominant in compact object astrophysics. This paper details the implications of this framework. For General Relativity, it implies a non-minimal coupling that violates the Equivalence Principle. For QCD, it implies a direct link between spacetime geometry and the chiral condensate. For nuclear physics, it predicts a "Dynamic Valley of Stability" and "Geometrically-Induced Fission" (GIF). For astrophysics, it is transformative for neutron star equations of state and provides a new, testable

fission channel (GIF) that would directly alter kilonova light curves. The theory is found to be internally consistent, observationally viable, and falsifiable, with kilonova nucleosynthesis and neutron star EoS constraints providing the primary future observational pathways.

1 The GIMV Hypothesis: A Formal Analysis

The central premise of the theoretical framework outlined in the foundational paper, "A 6DT-Stoke Framework for Geometrically-Induced Mass Variation (GIMV): Formalism and Application to Nuclear Stability" [1], represents a radical departure from the established principles of 20th-century physics. It challenges the notion of mass as an intrinsic, fixed property of a particle. This section deconstructs the GIMV hypothesis, its formal Lagrangian implementation, and its identification of a new coupling between matter and spacetime curvature.

1.1 From Kinematic Identity to Dynamic Principle

Modern physics holds rest mass to be conserved. In special and general relativity, the rest mass m_0 of a particle is a Poincaré-invariant scalar, representing irreducible energy content in the particle's rest frame. This mass is fixed and immutable. The GIMV framework, however, proposes a fundamental alteration: building on results from a posited six-dimensional vector-time (6DT) framework [2], it suggests rest mass is not constant but a dynamic scalar field.

The paper's starting point is the "Stoke-6DT" identity, $S_{6D} \equiv -c^2\, dm_0/d\tau$ [1,2]. This relates the work done by an "anomalous" (non-geodesic) force S_{6D} to a change in the particle's rest mass m_0 with respect to proper time τ. Previous work treated this as a mere kinematic identity (energy–mass conversion). The GIMV hypothesis is born by "elevating this... to a dynamic principle" [1].

This elevation is profound. If an external force can "do work" to change a particle's rest mass, then rest mass itself becomes a dynamic scalar field $m_0(x)$ influenced by environment [1]. The 6DT framework identifies the source of this anomalous force as the geometry of extra dimensions, which is sourced by the Hessian of the Newtonian potential $K_{ij} = \partial_i \partial_j \Phi$ (the classical tidal tensor). This yields a chain: $K_{ij} \leftrightarrow A_{\text{anom}} \leftrightarrow dm_0/d\tau$, implying a direct, non-perturbative coupling between the local gravitational tidal field and the particle's rest mass [1].

This is the core of GIMV: $m_0 = m_0(x, \mathcal{K})$, where $\mathcal{K}$ is some scalar invariant of the tidal field.

The theory focuses this hypothesis on the nucleon (proton and neutron), positing that the nucleon rest mass m_N is a function of the local gravitational environment [1].

1.2 Lagrangian Formalism: A Non-Minimal Coupling

To turn this principle into a predictive 4D field theory, the GIMV framework embeds it in a Lagrangian. A new **non-minimal coupling (NMC)** term is added to the standard Dirac Lagrangian for the nucleon field ψ_N [1]:

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}_N (i\gamma^\mu \nabla_\mu - m_N^0)\, \psi_N,$$

where m_N^0 is the bare nucleon mass (a constant) and ∇_μ is the covariant derivative ensuring general covariance [1]. GIMV introduces

$$\mathcal{L}_{\text{NMC}} = -\,\xi\, \mathcal{K}\, \bar{\psi}_N \psi_N,$$

so the total Lagrangian is $\mathcal{L} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{NMC}}$ [1].

This new term is a Lorentz scalar (since $\bar{\psi}\psi$ and $\mathcal{K}$ are scalars). The constant ξ quantifies its strength [1]. The consequence is that the mass term can be redefined:

$$\mathcal{L} = \bar{\psi}_N \big(i\gamma^\mu \nabla_\mu - [m_N^0 + \xi\, \mathcal{K}]\big) \psi_N,$$

implying an **effective, position-dependent nucleon mass** $m_N^{\text{eff}}(x)$ [1]:

$$m_N^{\text{eff}}(x) = m_N^0 + \xi\, \mathcal{K}(x).$$

The field equation becomes a modified Dirac equation:

$$(i\gamma^\mu \nabla_\mu - m_N^{\text{eff}}(x))\, \psi_N = 0,$$

providing a concrete field-theoretic basis for the GIMV hypothesis [1].

A dimensional analysis of ξ shows $\xi\, \mathcal{K}$ carries units of mass. The Newtonian tidal tensor $K_{ij} = \partial_i \partial_j \Phi$ has units of T^{-2} (since Φ has units L^2/T^2). The proposed invariant $\mathcal{K}_N = K_{ij}K^{ij}$ then has units T^{-4} [1]. Thus $[\xi] = [m_N]/[\mathcal{K}] = M \cdot T^4$ (in SI: $\text{kg} \cdot \text{s}^4$). Any nucleon mass change is $\Delta m_N = \xi\, \mathcal{K}$ [1].

This Lagrangian is an effective field theory. The Standard Model and QCD describe the nucleon mass as an emergent property of quarks and gluons, intrinsically tied to chiral symmetry breaking. The GIMV Lagrangian $\mathcal{L}_{\rm NMC}$ does not attempt to model that UV-complete picture. It posits a low-energy phenomenological coupling at the level of the composite nucleon field, describing the *effect* (mass variation) without specifying the *mechanism* (e.g. how curvature alters the QCD vacuum).

1.3 Identifying the Tidal Invariant $\mathcal{K}$ with the Kretschmann Scalar

Choosing the scalar invariant $\mathcal{K}$ is critical for the GIMV gravity interface. The theory begins from the Newtonian tidal tensor K_{ij} [1], but a fundamental theory must be generally covariant. Thus $\mathcal{K}$ must be a true invariant built from the tensors of General Relativity.

The paper [1] notes that the Newtonian K_{ij} is the weak-field limit of the gravitoelectric part of the Riemann curvature R_{abcd}. To form a scalar, one can contract the Riemann tensor with itself. The simplest nontrivial invariant is the **Kretschmann scalar** K [3]:

$$K = R_{abcd}R^{abcd}.$$

The Kretschmann scalar is a measure of actual spacetime curvature. Unlike the Ricci scalar R (zero in vacuum outside a star), K remains nonzero in vacuum and is used to identify true singularities (where $K \to \infty$) [3].

The GIMV paper [1] proposes identifying its tidal invariant $\mathcal{K}$ with the Kretschmann scalar K. It validates this via two consistency checks:

1. **Physical Meaning:** Newtonian K_{ij} is the weak-field limit of R_{abcd}. Setting $\mathcal{K} = K$ ensures the theory reduces to its Newtonian motivation in the weak-field regime [1].

2. **Dimensional Consistency:** For a Schwarzschild spacetime of mass M, K is given by [1]:

$$K = \frac{48\, G^2 M^2}{c^4 r^6}.$$

   In SI units:

$$[K] \sim \frac{(L^3 M^{-1} T^{-2})^2 M^2}{(L/T)^4 L^6} \sim T^{-4},$$

matching the units of $\mathcal{K}_N$.

Thus GIMV sets $\mathcal{K} = K/48$ [1], giving the concrete coupling invariant (for Schwarzschild):

$$\mathcal{K}(r) = \frac{G^2 M^2}{c^4 r^6}.$$

This forms the basis for quantitative astrophysical predictions in GIMV [1].

2 Contextualizing GIMV in Fundamental Physics

The GIMV hypothesis, encoded by $\mathcal{L}_{\text{NMC}}$, is not standalone; it touches the pillars of fundamental physics. It must be situated relative to the Standard Model (SM), QCD, quantum field theory in curved spacetime (QFTCS), and General Relativity (GR).

2.1 Standard Model: Mass, the Higgs Mechanism, and BSM Physics

In the Standard Model, fundamental fermion masses are generated by the Higgs mechanism [4]. A Yukawa coupling between a fermion and the Higgs field yields a mass term $m_f = y_f v$ when the Higgs acquires its vacuum expectation value (VEV) $v \approx 174$ GeV [5]. In the SM, v is a universal constant of nature.

GIMV, $m_N = m_N(\mathcal{K})$, fundamentally challenges this. It represents beyond-Standard-Model (BSM) physics, interpretable in two ways:

1. **Interpretation A: Variable Higgs VEV.** The GIMV coupling could effectively mean the Higgs VEV v itself varies with curvature: $v = v(\mathcal{K})$. Then *all* particle masses (m_e, m_u, m_d, etc.) would vary with the tidal field.

2. **Interpretation B: Direct QCD–Gravity Coupling.** Note that nucleon mass m_N is *not* a fundamental mass; only $\sim 1\%$ comes from Higgs (quark masses), the rest $\sim 99\%$ is QCD binding energy. $\mathcal{L}_{\text{NMC}} = -\xi\mathcal{K}\,\bar{\psi}_N\psi_N$ [1] couples to the composite nucleon field, suggesting quark masses (Higgs) stay constant but the dominant QCD-generated portion of m_N has a new coupling to gravity, bypassing the Higgs mechanism.

The GIMV paper [1] focuses on the nucleon field, favoring (B). This implies a new "fifth force" coupling the nucleon's strong-interaction mass to spacetime curvature.

2.2 QCD and Chiral Symmetry

This is perhaps the most profound implication. Roughly 99% of the nucleon mass is not from the Higgs at all, but from the non-perturbative dynamics of QCD — specifically, the **spontaneous breaking of chiral symmetry** [6]. In vacuum, the QCD Lagrangian has an approximate chiral symmetry (independent left- and right-handed quarks) that is spontaneously broken; the order parameter is the **chiral condensate** $\Sigma = \langle\bar{\psi}\psi\rangle$, which attains a large value [7]. The nucleon mass is, to a good approximation, proportional to $\Sigma^{1/3}$.

If GIMV is correct and $m_N = m_N(\mathcal{K})$, then the QCD vacuum itself must become sensitive to the local tidal field:

$$\Sigma = \Sigma(\mathcal{K}).$$

In a region of high curvature (e.g. a neutron star surface), the condensate's value is altered. This is a *new* mechanism for chiral symmetry modification. Conventionally, chiral symmetry is partially restored only at extreme *density* or *temperature* (e.g. in neutron star cores or the early universe) [8]. GIMV posits a third mechanism: modification by gravitational curvature alone. It suggests the strong-force vacuum is fundamentally linked to spacetime geometry.

2.3 A Non-Standard QFT in Curved Spacetime

The GIMV Lagrangian $\mathcal{L} = \bar{\psi}_N[i\gamma^\mu\nabla_\mu - (m_N^0 + \xi\mathcal{K})]\psi_N$ [1] can be viewed as a quantum field theory in curved spacetime (QFTCS). Standard QFTCS deals with fields on a fixed curved background $g_{\mu\nu}$. However, GIMV is a *non-minimal* QFTCS: in standard QFTCS, one employs minimal coupling (the "comma-goes-to-semicolon" rule), where interaction terms are as in flat spacetime but derivatives are replaced by covariant derivatives ∇_μ. GIMV adds a new non-minimal term coupling the matter field directly to a curvature invariant ($K = R_{abcd}R^{abcd}$).

This leads to different physics. Standard QFTCS predicts particle creation (e.g. Hawking or Unruh radiation, pair production in an expanding universe) when the gravitational field is *time-dependent* or has a horizon. GIMV, by contrast, predicts a change in a *particle's intrinsic mass* even in a *static* field (e.g. outside a static star or black hole), as long as there is tidal curvature $\mathcal{K}$ [1].

2.4 Equivalence Principle (EP)

The Equivalence Principle is central to GR. The Weak EP states that a test body's trajectory in a gravitational field is independent of its composition. This follows from the equality of inertial and gravitational mass $m_i = m_g$ [9].

The GIMV hypothesis $m_N^{\mathrm{eff}}(x) = m_N^0 + \xi\mathcal{K}(x)$ [1] is a direct violation of EP [10]. The total mass-energy of an atom is $M = Zm_p + Nm_n - B(A,Z)/c^2$. In GIMV:

$$M(\mathcal{K}) = Z(m_p^0 + \xi\mathcal{K}) + N(m_n^0 + \xi\mathcal{K}) - B(A,Z;\mathcal{K})/c^2,$$

$$M(\mathcal{K}) = M^0 + A\,\xi\,\mathcal{K} - \delta B(\mathcal{K})/c^2.$$

As shown in [1], the binding energy B also becomes a function of $\mathcal{K}$ (since the SEMF coefficients become $a_i(\mathcal{K})$). Because binding fractions differ between elements (e.g. Al vs Pt), $M(\mathcal{K})$ acquires a composition-dependent term.

Thus two bodies of different composition, in the same gravity field, would experience slightly different accelerations — a "fifth force" that EP experiments search for [10]. The null results of high-precision Eötvös tests (constraining $\Delta m_N/m_N < 10^{-14}$ in Earth's field) then impose an upper bound: $\xi < 7.3 \times 10^{-30}$ kg·s^4 [1]. This is the lynchpin of GIMV: ξ *must* be small enough for the theory to be hidden in weak fields.

2.5 Table 1: GIMV vs. Standard Physics

Table 1 contrasts the GIMV paradigm with standard physics principles:

3 Distinguishing GIMV from Other Variable-Mass Paradigms

The idea of "variable mass" is not new. However, the *mechanism* in GIMV (coupling to local curvature $\mathcal{K}$) is fundamentally distinct from other scenarios of varying mass or constants.

3.1 Distinction 1: Gravitational Coupling vs. In-Medium Modification

In standard nuclear physics, a "variable nucleon mass" is well known as an **in-medium effective mass** M^*. But M^* varies with the nuclear medium's *density* (ρ) and *temperature* (T), not gravity.

Table 1: GIMV vs. Standard Physics: Foundational Principles

	Standard Physics (SM + GR)	**GIMV Framework**
Nucleon Mass m_N	Fundamental constant (emergent from QCD) [6]	Dynamic scalar field: $m_N^{\rm eff}(x) = m_N^0 + \xi\mathcal{K}(x)$ [1]
Source of Mass	Higgs mechanism (for quarks) + QCD chiral condensate (for binding) [4,6]	New non-minimal coupling of composite nucleon field to curvature (gravity) [1]
Equivalence Principle	Inviolable: $m_i = m_g$ (composition-independent) [9]	Violated: mass depends on composition and position (tidal field) [1]
Field Coupling	*Minimal coupling:* matter fields couple only to metric $g_{\mu\nu}$	*Non-minimal coupling:* matter fields couple to curvature invariant $K = R_{abcd}R^{abcd}$ [1]
Testable Domain	All regimes (no exceptions)	Negligible in terrestrial (weak-field); dominant in strong-field (high $\mathcal{K}$) [1]

For example, in relativistic mean-field (RMF) models, the nucleon mass is reduced by a strong scalar meson field σ: $M^* = M - g_\sigma\sigma$, where σ depends on the local nucleon density [11]. This density-dependent mass $M^*(\rho)$ is crucial to nuclear saturation and underlies modern neutron star EoS models [11]. Such in-medium mass modification is also a key explanation of the EMC effect (the apparent modification of nucleon structure inside nuclei) [12].

GIMV, in contrast, is entirely different. The mass variation $m_N(\mathcal{K})$ is driven by the *external* gravitational tidal field, irrespective of local density ρ.

This leads to a new prediction. In a neutron star, *both* effects are present. The total effective nucleon mass would depend on density and curvature:

$$m_N^{\rm total}(r) = M^*(\rho(r), T(r)) + \xi\,\mathcal{K}(r).$$

Thus a neutron star's EoS depends not only on its density profile (as in GR) but also on its own curvature field. This is a new, untested phenomenology that would alter all neutron star models.

3.2 Distinction 2: Local Spatial Variation vs. Cosmological Time Variation

GIMV must also be distinguished from cosmological "varying constant" theories. Many BSM theories (e.g. some Kaluza–Klein or string models) propose that fundamental constants (like the fine-structure constant α or G) vary over *cosmological time* [13].

In such models, constants are often linked to a rolling scalar field (a *dilaton/quintessence* field ϕ). As the universe expands, $\phi(t)$ slowly evolves, causing constants to drift in time, e.g. $\alpha(t) = \alpha_0 \, F[\phi(t)]$ [14]. These theories are constrained by comparing constants today (atomic clock experiments) vs the distant past (quasar absorption spectra or BBN) [15].

GIMV is fundamentally different: not a time variation but a **local spatial** variation driven by the local curvature $\mathcal{K}(x)$. In GIMV, the nucleon mass is constant on Earth (since $\mathcal{K}_{Earth}$ is fixed) but has a different constant value near a neutron star (where $\mathcal{K}_{NS}$ is huge). m_N would only change in time if the *local* curvature source changed (e.g. during a neutron star merger). This is a much more localized and testable prediction than a slow cosmological drift.

4 Core Consequences: Gravitationally-Modulated Nuclear Physics

The predictive power of GIMV comes from applying $m_N(\mathcal{K})$ to fundamental nuclear physics equations. The paper [1] derives the consequences for the **semi-empirical mass formula (SEMF)**, the liquid-drop model for nuclear binding energy.

4.1 The GIMV-Modified SEMF

The SEMF for a nucleus of mass number A and proton number Z is:

$$B(A, Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A - 2Z)^2}{A} + \dots$$

The coefficients a_V (volume), a_S (surface), a_C (Coulomb), and a_A (asymmetry) are empirical parameters arising from the nuclear forces [16].

GIMV ($m_N \to m_N(\mathcal{K})$) implies these coefficients, which depend on m_N, must themselves become functions of the tidal field: $a_i \to a_i(\mathcal{K})$ [1]. The following sections re-derive and validate this logic.

4.2 Derivation 1: $m_N(\mathcal{K})$ and the Asymmetry Term a_A

The asymmetry term a_A is a quantum effect of the Pauli principle. A nucleus with $N \neq Z$ (excess neutrons) must place the extra neutrons in higher energy states, increasing kinetic energy and reducing binding [1].

The GIMV paper [1] uses a standard **Fermi gas model**:

1. Treat protons and neutrons as separate Fermi gases in a volume $V \propto r_0^3 A$ (with r_0 the nuclear radius parameter).

2. Fermi energy $E_F \sim p_F^2/(2m_N) \propto n^{2/3}/m_N$, where n is number density.

3. Total kinetic energy $E_{\text{kin}} \sim N E_{F,n} + Z E_{F,p}$.

4. Expanding for small asymmetry $(N - Z)$ yields the SEMF form, identifying the coefficient of $(N - Z)^2/A$ as a_A.

This derivation [1] shows a_A depends inversely on m_N (and on r_0, which sets density):

$$a_A \propto \frac{1}{m_N\, r_0^2}.$$

Thus a change in nucleon mass $m_N(\mathcal{K})$ must induce a change in $a_A(\mathcal{K})$.

4.3 Derivation 2: The Critical $m_N \to r_0$ Link

The surface (a_S) and Coulomb (a_C) terms depend on geometry (radius r_0) rather than directly on m_N [1]: - $a_C \propto 1/r_0$ (from Coulomb energy of a sphere). - $a_S \propto r_0^2$ (from surface area).

So the GIMV model needs a link showing that a change in m_N causes a change in r_0. The paper [1] identifies this "missing link" and cites **chiral EFT** results on how nuclear saturation properties vary with fundamental constants. The key relation is:

$$\frac{\delta r_0}{r_0} = K_\pi \frac{\delta m_\pi}{m_\pi} + K_N \frac{\delta m_N}{m_N},$$

with sensitivity coefficients $K_\pi \approx +1.8$ and $K_N \approx -4.8$. In GIMV, $\delta m_\pi = 0$ (since the coupling is to the nucleon field), so:

$$\frac{\delta r_0}{r_0} \approx -4.8\, \frac{\delta m_N}{m_N}.$$

Physically, an increase in m_N ($\delta m_N > 0$) lowers nucleon kinetic energy, allowing nuclei to bind more tightly and at smaller radius ($\delta r_0 < 0$) [17].

Independent work confirms this. Studies of nuclear structure under varying constants [17] explicitly find "$K_\pi = +1.8$, $K_N = -4.8$" [18]. This shows the GIMV paper's missing link rests on established chiral EFT calculations [19], making the subsequent derivations internally consistent.

4.4 Derivation 3: Cooperative Scaling of a_S and a_C

With the $m_N \to r_0$ coupling in hand, the GIMV-modified SEMF can be obtained. Define the fractional mass shift $\epsilon_\mathcal{K} \equiv \delta m_N/m_N = (\xi/m_N^0)\mathcal{K}$ [1].

- **Radius change:** $\dfrac{\delta r_0}{r_0} \approx -4.8\,\epsilon_\mathcal{K}$.
- a_C **scaling** ($\propto r_0^{-1}$)**:** $\dfrac{\delta a_C}{a_C} = -\dfrac{\delta r_0}{r_0} \approx +4.8\,\epsilon_\mathcal{K}$. (Coulomb repulsion *increases* with $\mathcal{K}$.) Thus $a_C(\mathcal{K}) \approx a_C^0(1 + 4.8\,\epsilon_\mathcal{K})$.
- a_S **scaling** ($\propto r_0^2$)**:** $\dfrac{\delta a_S}{a_S} = 2\,\dfrac{\delta r_0}{r_0} \approx -9.6\,\epsilon_\mathcal{K}$. (Surface tension *decreases* with $\mathcal{K}$.) Thus $a_S(\mathcal{K}) \approx a_S^0(1 - 9.6\,\epsilon_\mathcal{K})$.
- a_A **scaling** ($\propto (m_N r_0^2)^{-1}$)**:** $\dfrac{\delta a_A}{a_A} = -\dfrac{\delta m_N}{m_N} - 2\,\dfrac{\delta r_0}{r_0} \approx -\epsilon_\mathcal{K} + 9.6\,\epsilon_\mathcal{K} = +8.6\,\epsilon_\mathcal{K}$. (The asymmetry term *increases*.) Thus $a_A(\mathcal{K}) \approx a_A^0(1 + 8.6\,\epsilon_\mathcal{K})$.

These results, summarized in Table 2, are the central theoretical outcome of GIMV [1].

Table 2: Scaling of SEMF coefficients with tidal field $\mathcal{K}$ (for small fractional mass shift $\epsilon_\mathcal{K} = \xi\mathcal{K}/m_N^0$).

Coefficient	**Depends on**	**Scale ($\delta a_i/a_i^0$ per $\delta m_N/m_N$)**	**Modified form $a_i(\mathcal{K})$**
a_S (Surface)	r_0^2	-9.6	$a_S^0\big(1 - 9.6\,\epsilon_\mathcal{K}\big)$
a_C (Coulomb)	r_0^{-1}	$+4.8$	$a_C^0\big(1 + 4.8\,\epsilon_\mathcal{K}\big)$
a_A (Asymmetry)	$(m_N r_0^2)^{-1}$	$+8.6$	$a_A^0\big(1 + 8.6\,\epsilon_\mathcal{K}\big)$

The most important consequence is the cooperative destabilization of heavy nuclei under strong curvature: the surface term (nuclear glue) *weakens* while the Coulomb term (electrostatic repulsion) *strengthens*.

4.5 Prediction 1: A Dynamic Valley of Stability

One consequence of the $a_i(\mathcal{K})$ coefficients is that the line of beta-stable nuclei (the valley of stability) shifts in a strong tidal field [1].

The center of stability Z_{stable} (the proton number minimizing mass $M(A, Z)$ at fixed A) is given in the standard case by:

$$Z_{\text{stable}}(0) \approx \frac{A}{2} \frac{1}{1 + \frac{a_C A^{2/3}}{4\, a_A}}.$$

In GIMV, replacing a_C and a_A with their $\mathcal{K}$-dependent forms (to first order in $\epsilon_{\mathcal{K}}$) yields [1]:

$$Z_{\text{stable}}(\mathcal{K}) \approx \frac{A}{2} \frac{1}{1 + \frac{a_C^0 (1+4.8\, \epsilon_{\mathcal{K}}) A^{2/3}}{4\, a_A^0 (1+8.6\, \epsilon_{\mathcal{K}})}}.$$

Consequence: For $\xi > 0$, the asymmetry term grows faster than the Coulomb term (+8.6% vs +4.8% per $\epsilon_{\mathcal{K}}$). Thus a_C/a_A *decreases*, moving Z_{stable} closer to $A/2$ (i.e. shifting stability toward $N = Z$), as illustrated in Fig. 2 of [1].

This predicts that a nucleus stable on Earth (e.g. a neutron-rich isotope) could become beta-unstable in a high-$\mathcal{K}$ environment. A new, gravitationally-induced decay path would open in extreme astrophysical sites [1].

4.6 Prediction 2: Geometrically-Induced Fission (GIF)

The most dramatic prediction of GIMV is **Geometrically-Induced Fission (GIF)** [1]. This arises from the combined weakening of surface binding and strengthening of Coulomb repulsion.

Nuclear fission is a competition between nuclear attraction (surface term E_S) and electrostatic repulsion (Coulomb term E_C) [1]: - $E_S \propto a_S A^{2/3}$ (short-range glue). - $E_C \propto a_C Z^2/A^{1/3}$ (long-range repulsion).

A nucleus's stability against spontaneous fission is measured by the **fissility parameter** x, the ratio of these energies [1]:

$$x \equiv \frac{E_C}{2E_S} = \left(\frac{a_C}{2a_S}\right) \frac{Z^2}{A}.$$

The fission barrier B_f (energy needed to split the nucleus) roughly scales as $(1 - x)$. If $x < 1$, a nucleus has a barrier (e.g. ^{238}U, $x \approx 0.78$) and is stable against spontaneous fission. If $x \geq 1$, the barrier vanishes ($B_f \leq 0$) and the nucleus fissions promptly (no stable nucleus beyond $Z \sim 104$).

In GIMV, x becomes a function of $\mathcal{K}$. Inserting the $a_i(\mathcal{K})$ from Table 2, the paper derives [1]:

$$x(\mathcal{K}) = \left(\frac{a_C(\mathcal{K})}{2a_S(\mathcal{K})}\right) \frac{Z^2}{A} \;\approx\; \left(\frac{a_C^0 (1 + 4.8\, \epsilon_{\mathcal{K}})}{2a_S^0 (1 - 9.6\, \epsilon_{\mathcal{K}})}\right) \frac{Z^2}{A} \;\approx\; x(0)\, \frac{1 + 4.8\, \epsilon_{\mathcal{K}}}{1 - 9.6\, \epsilon_{\mathcal{K}}}.$$

This shows a potent one-two punch on stability (for $\xi > 0$): (1) the numerator increases (Coulomb grows), and (2) the denominator decreases (surface shrinks). Together, they drive $x(\mathcal{K}) \to 1$. A nucleus like ^{238}U, stable on Earth, placed in a strong tidal field could have its fissility pushed past unity and undergo **spontaneous GIF** [1].

5 Astrophysical Implications I: Stellar Structure and Compact Objects

To test GIMV, we turn to cosmic laboratories. Its viability as a "strong-field-only" theory depends on $\mathcal{K}$ in various astrophysical settings.

5.1 Impact on Main-Sequence Stars

Normal stars (main-sequence or giants) are governed by hydrostatic equilibrium (balance of gravity and fusion pressure). A variable m_N would, in principle, alter the stellar equation of state and nuclear reaction rates.

However, quantitative estimates show GIMV is irrelevant for ordinary stars. The tidal invariant $\mathcal{K}$ in these stars is extremely small. Using $\mathcal{K}(r) = G^2M^2/(c^4r^6)$ [1]: - Earth (surface): $\mathcal{K}_{Earth} \sim 2.3\times10^{-12}$ s^{-4} [1]. - Sun (surface): $M \approx 2.0\times10^{30}$ kg, $r \approx 7.0\times10^8$ m gives $\mathcal{K}_{Sun} \sim 1.8\times10^{-13}$ s^{-4} (calc.).

The Sun's tidal field is an order of magnitude weaker than Earth's. Red giants (larger r) and white dwarfs (smaller M/r) have even lower $\mathcal{K}$. Thus GIMV has *no* observable impact on normal stars, their lifetimes, or white dwarf structure.

5.2 Neutron Star Equation of State (EoS)

GIMV becomes significant only in the most extreme gravity: compact objects. A neutron star's structure is set by its EoS $P(\rho)$ (pressure vs density) [11].

As noted in Section 3.1, standard RMF models already have a density-dependent effective mass $M^*(\rho)$. GIMV adds an independent curvature dependence $\mathcal{K}(r)$. The EoS becomes $P(\rho, \mathcal{K})$.

At the surface of a 1.4 $M_\odot$ neutron star of radius 10 km: - Neutron Star (surface): $M \approx 2.8\times10^{30}$

kg, $r \approx 10^4$ m.

$$\mathcal{K}_{NS} \sim \frac{(6.67 \times 10^{-11})^2 (2.8 \times 10^{30})^2}{(3 \times 10^8)^4 (10^4)^6} \sim 3.4 \times 10^{22}\ \mathrm{s}^{-4}.$$

This value (as given in [1]) is ~ 34 orders of magnitude larger than Earth's. Here, $\xi\mathcal{K}$ is no longer negligible but can dominate the nucleon mass.

5.3 Neutron Star Mass–Radius and the TOV Limit

Solving the Tolman–Oppenheimer–Volkoff (TOV) equations with a given EoS yields the neutron star mass–radius relation. A GIMV-modified EoS $P(\rho, \mathcal{K})$ gives a different curve.

The effect depends on the sign of ξ. A "stiffer" EoS (more pressure at given ρ) supports a larger max mass $M_{\max}$ and yields larger radii at a given mass. EoS stiffness correlates with the nucleon effective mass M^*: a lower M^* typically gives a stiffer EoS.

The GIMV paper [1] assumes $\xi > 0$ (for GIF predictions). $\delta m_N = \xi\mathcal{K} > 0$ means nucleon mass *increases* in strong fields, leading to a *softer* EoS (higher M^*).

A softer EoS lowers the maximum neutron star mass $M_{\max}$ (TOV limit) supportable against collapse. Observations of massive pulsars like PSR J0740+6620 ($M \approx 2.1\,M_\odot$) already rule out overly soft EoS [20]. Thus ξ (if positive) cannot be so large as to soften the EoS below $2\,M_\odot$. The mere existence of $\sim 2\,M_\odot$ pulsars provides an independent astrophysical upper bound on ξ (not considered in [1]). Conversely, $\xi < 0$ would stiffen the EoS and raise $M_{\max}$.

5.4 Table 3: $\mathcal{K}$ and GIMV Effects in Key Environments

Table 3 compiles $\mathcal{K}$ values in various environments (from [1] and our calculations), and the resulting fractional mass shifts using the terrestrial bound $\xi_{EP} < 7.3 \times 10^{-30}$ kg·s^4.

Table 3: Tidal invariant $\mathcal{K}$ and GIMV effect in selected environments. ξ_{EP} is the Earth EP bound [1].

Environment	$\mathcal{K}$ (s^{-4})	ξ for 1 MeV shift (kg·s^4)	Max $\Delta m_N/m_N$ (with ξ_{EP})	**GIMV Impact**
Earth (surface)	$\sim 2.3 \times 10^{-12}$ [1]	$\sim 7.7 \times 10^{-19}$ [1]	$< 10^{-14}$ (by defn)	Negligible
Sun (surface)	$\sim 1.8 \times 10^{-13}$ (calc.)	$\sim 9.9 \times 10^{-18}$ (calc.)	$< 10^{-15}$	Negligible
BBN epoch ($t \sim 10$ s)	$\sim 3.1 \times 10^{-6}$ (calc.)	$\sim 5.7 \times 10^{-25}$ (calc.)	$< 1.3 \times 10^{-8}$	Constraining
10 $M_\odot$ BH (ISCO)	$\sim 3.6 \times 10^{12}$ [1]	$\sim 4.9 \times 10^{-43}$ [1]	$\sim 1.6\%$	Significant
1.4 $M_\odot$ NS (surface)	$\sim 3.4 \times 10^{22}$ [1]	$\sim 5.2 \times 10^{-53}$ [1]	~ 150 ($> 10000\%$)	Dominant

This table quantitatively justifies the "strong-field-only" conclusion. The EP bound $\xi < 7.3 \times 10^{-30}$ kg·s^4 permits $< 10^{-14}$ fractional mass shift on Earth. Yet the same ξ would cause $\Delta m_N \sim$

$150\, m_N^0$ at a neutron star surface — clearly impossible. Thus the *actual* ξ must be much smaller than the EP limit.

6 Astrophysical Implications II: Nucleosynthesis and Observables

The best tests of GIMV may lie in cataclysmic events that combine strong gravity and nuclear physics: neutron star mergers (kilonovae) and the early universe.

6.1 The r-Process in Neutron Star Mergers

The **r-process** (rapid neutron capture) is responsible for producing the heaviest elements (e.g. gold, uranium, plutonium) in the universe [24]. It requires an environment with an enormous flux of free neutrons, such as the ejecta from a binary neutron star (BNS) merger [22].

In a BNS merger, the r-process path is governed by competition among: 1. **Neutron Capture (n, γ):** pushes nuclei to higher A (more neutron-rich). 2. **Beta Decay (β^-):** increases the proton number Z. 3. **Photodissociation (γ, n):** knocks out neutrons (stalling at magic numbers). 4. **Fission:** splits very heavy nuclei ($A \gtrsim 250$), recycling material [23].

The final r-process abundances are extremely sensitive to nuclear input physics. Nuclear masses (which set S_n and thus the path) and fission barriers/yields (which determine recycling) are major uncertainties [23].

GIMV adds a new explosive ingredient. The kilonova ejecta (the neutron-rich merger debris) is not in a static field — it is expanding relativistically (0.1–0.3c) through the *immense*, rapidly changing tidal field $\mathcal{K}(t)$ of the remnant (either a hypermassive neutron star or a nascent black hole). Here, $\mathcal{K}$ can far exceed even a static NS's value.

This is the ideal laboratory for **Geometrically-Induced Fission (GIF)**. GIMV predicts the extreme tidal field will destabilize heavy nuclei, driving fissility $x(\mathcal{K})$ past unity. This opens a new fission channel not in current r-process models. Nuclei deemed stable against fission under normal conditions would fission due to gravity, dramatically altering the r-process path and final abundances.

6.2 Kilonova Light Curves: A Direct Probe of GIF

This new fission channel has a direct observational signature: the **kilonova** light curve. A kilonova (e.g. from GW170817) is an optical/IR transient powered by the radioactive decay of freshly synthesized r-process nuclei in the ejecta [22].

The brightness and color evolution of a kilonova depend on two key factors: 1. The **radioactive heating rate** $\epsilon(t)$: energy per unit time from nuclear decays, which depends on the precise mix of nuclei (especially fission yields) [23]. 2. The **opacity**: determined by the presence of heavy (lanthanide/actinide) elements.

GIF would alter $\epsilon(t)$. By forcing fission in nuclei that normally wouldn't fission, and by changing fission fragment yields, GIMV would produce a *different* distribution of radioactivities. This implies a *unique* heating rate $\epsilon_{\text{GIMV}}(t)$. The late-time (days–weeks) kilonova light curve — thought to be powered partly by the spontaneous fission of ^{254}Cf (half-life 60.5 d) [25] — would be especially sensitive to an extra fission channel.

This provides a smoking gun for GIMV. If future kilonovae (discovered via GW triggers and follow-up telescopes) show light curves that *cannot* be explained by standard r-process models but *can* be matched by models including GIMV $a_i(\mathcal{K})$ coefficients and the GIF channel, it would be the first direct evidence for this new physics. Indeed, the GIMV paper [1] suggests kilonova nucleosynthesis as a primary way to constrain ξ.

6.3 Constraint 1: Equivalence Principle Experiments

The GIMV paper [1] first establishes viability by using Equivalence Principle (EP) experiments to bound ξ: - Limit: Eötvös experiments constrain composition-dependent accelerations to $\Delta a/a < 10^{-13}$–10^{-14}, implying $\Delta m_N/m_N < 10^{-14}$ in Earth's field [1]. - Earth's field: $\mathcal{K}_{Earth} \approx 2.3 \times 10^{-12}\ \text{s}^{-4}$ [1]. - Inference: Using $\Delta m_N \approx \xi\mathcal{K}$,

$$\xi\,\mathcal{K}_{Earth} < 10^{-14}\,m_N^0,$$

$$\xi_{EP} < \frac{10^{-14} \times 1.67 \times 10^{-27}\ \text{kg}}{2.3 \times 10^{-12}\ \text{s}^{-4}} \sim 7 \times 10^{-30}\ \text{kg}{\cdot}\text{s}^4.$$

- Result: $\xi < 7.3 \times 10^{-30}\ \text{kg}{\cdot}\text{s}^4$ [1].

This bound ensures GIMV is completely negligible in the solar system (a must for any modified gravity theory).

6.4 Constraint 2: Big Bang Nucleosynthesis (BBN)

A crucial laboratory for GIMV (overlooked in [1]) is **Big Bang Nucleosynthesis (BBN)**. BBN occurred in the first few minutes of the universe ($t \sim 10$–200 s) and precisely predicts the primordial abundances of light elements (D, ^{3}He, ^{4}He, ^{7}Li) [26]. BBN is extremely sensitive to the values of fundamental constants and nuclear parameters [26], giving another constraint on ξ.

Step 1: Effect on n–p mass difference Q. BBN yields (especially ^{4}He) are very sensitive to the neutron-proton mass difference $Q = m_n - m_p \approx 1.293$ MeV, which sets the n/p ratio at freeze-out. GIMV's coupling is isospin-blind (acts equally on m_n and m_p), so $\delta m_n = \delta m_p = \xi\mathcal{K}$. Thus $\delta Q = 0$. The crucial parameter Q is unchanged — GIMV passes this stringent test.

Step 2: Effect on m_N and binding energies. BBN is also sensitive to the absolute m_N and key binding energies (e.g. deuteron binding B_d). A varying $m_N(\mathcal{K})$ would shift these. To test this, estimate $\mathcal{K}$ during BBN.

Step 3: $\mathcal{K}$ during BBN. In radiation-dominated FLRW cosmology ($a(t) \propto t^{1/2}$), the Kretschmann scalar is $K = 3/(2t^4)$. Hence $\mathcal{K} = K/48 = 1/(32t^4)$ [27]. At $t \approx 10$ s (deuterium formation time [26]):

$$\mathcal{K}_{BBN}(10\text{ s}) \sim \frac{1}{32\,(10\text{ s})^4} = 3.1 \times 10^{-6}\text{ s}^{-4}.$$

Step 4: BBN bound on ξ. This $\mathcal{K}$ is 10^6 times Earth's. BBN predictions match observations to $\sim 1\%$, so demand $\Delta m_N/m_N < 10^{-8}$ during BBN:

$$\xi\,\mathcal{K}_{BBN} < 10^{-8}\,m_N^0,$$

$$\xi_{BBN} < \frac{10^{-8} \times 1.67 \times 10^{-27}\text{ kg}}{3.1 \times 10^{-6}\text{ s}^{-4}} \approx 5.4 \times 10^{-30}\text{ kg}\cdot\text{s}^4.$$

This BBN bound is slightly tighter (by $\sim 25\%$) than the EP bound (7.3×10^{-30}). Our analysis thus *confirms* GIMV's viability in known physics, while narrowing ξ's allowed range. The "viability gulf" still yawns: the ξ needed for a 1 MeV mass shift at a neutron star ($\sim 10^{-53}$) is 23 orders below even the BBN bound.

7 Synthesis and Conclusion

We have treated the Geometrically-Induced Mass Variation (GIMV) framework [1] as a serious, testable proposal. The idea that nucleon mass couples to the gravitational tidal invariant $\mathcal{K}$ has far-reaching implications across physics.

7.1 Viability: A Strong-Field-Only Phenomenon

Our analysis supports the GIMV paper's [1] conclusion: the effect can only appear in extremely strong curvature. The $\sim$ 34 orders-of-magnitude gap in $\mathcal{K}$ between Earth ($\sim 10^{-12}$ s^{-4}) and a neutron star ($\sim 10^{22}$ s^{-4}) [1] acts as a natural shield. This allows ξ to be small enough to evade all terrestrial tests, yet large enough to dominate in compact-object environments [1].

We strengthened this by computing a BBN bound $\xi < 5.3 \times 10^{-30}$ kg·s^4, slightly tighter than the EP bound. Even this is astronomically above the $\xi \sim 10^{-53}$ kg·s^4 needed to cause a 1 MeV shift at a neutron star [1]. The theory is not excluded; it is simply pushed deep into the strong-field regime. Moreover, our independent validation of the $m_N \to r_0$ link ($K_N = -4.8$) [17,18] confirms that GIMV's nuclear derivations rest on published (though specialized) chiral EFT results [19].

7.2 Implications for All of Physics

In summary, GIMV implies: - **General Relativity:** A modification with non-minimal coupling $\mathcal{L}_{NMC} = -\xi\mathcal{K}\,\bar{\psi}_N\psi_N$ [1]. This explicitly violates the Equivalence Principle, making mass composition- and position-dependent [10]. - **Standard Model & QCD:** A BSM link between geometry and the QCD vacuum. It implies the chiral condensate (which generates $\sim$ 99% of m_N) [6] becomes function of the Kretschmann scalar. - **Nuclear Physics:** All nuclear properties become environment-dependent. SEMF coefficients a_i become $a_i(\mathcal{K})$ [1]. Two key predictions: a **Dynamic Valley of Stability** shifting toward $N = Z$ in strong fields [1], and **Geometrically-Induced Fission** (GIF) whereby a nucleus stable in weak gravity fissions in strong gravity [1]. - **Astrophysics:** Negligible for normal stars; transformative for compact objects. It adds a curvature term to the neutron star EoS, altering mass–radius and giving a new ξ constraint from $2M_\odot$ pulsars [20]. Critically, it introduces GIF in the r-process, altering kilonova nucleosynthesis and light curves [21].

7.3 Future Observational Tests

GIMV is highly falsifiable. We highlight two pathways: 1. **Kilonova nucleosynthesis (the "smoking gun"):** Run r-process network simulations with GIMV $a_i(\mathcal{K})$ and GIF. These will produce distinct abundance patterns and light curves $\epsilon_{\mathrm{GIMV}}(t)$ [23]. Comparing to future kilonova observations (from BNS mergers detected by LIGO/Virgo/KAGRA) can directly test or measure ξ. 2. **Neutron star EoS constraints ("precision bound"):** As neutron star observations improve (e.g. NICER radii, precise pulsar masses), the M–R relation and $M_{\max}$ will be tightly constrained. These can set an upper limit on ξ independent of other tests. A positive ξ softens the EoS, and too large a ξ would be ruled out by existing $2M_\odot$ pulsars.

8 Conclusion

This analysis has treated the Geometrically-Induced Mass Variation (GIMV) framework [1] as a serious, internally consistent, and testable physical hypothesis. Our exhaustive review confirms that GIMV's central premise—a non-minimal coupling of the composite nucleon mass to the gravitational Kretschmann scalar $\mathcal{K}$—is a viable "strong-field-only" phenomenon. The theory's ability to survive rests entirely on the "viability gulf": the ~ 34 order-of-magnitude difference in $\mathcal{K}$ between terrestrial environments and the surface of a neutron star. This gulf allows the coupling constant ξ to be negligible for Equivalence Principle (EP) tests while remaining dominant in compact object astrophysics. We have strengthened this viability claim by deriving a new constraint from Big Bang Nucleosynthesis, $\xi < 5.4 \times 10^{-30}$ kg·s^4, which is consistent with and slightly more stringent than the terrestrial EP bound.The implications of this framework, if correct, are profound, touching every pillar of modern physics. For General Relativity, it represents a well-defined, non-minimal modification that explicitly violates the Equivalence Principle. For quantum chromodynamics, it posits a radical, new BSM link between spacetime geometry and the strong force, implying that the QCD chiral condensate itself is a function of the local tidal field, $\Sigma = \Sigma(\mathcal{K})$. Phenomenologically, GIMV's power lies in its concrete, testable predictions for nuclear physics. By validating the $m_N \to r_0$ link [17, 18], we affirmed the paper's [1] derivation of $\mathcal{K}$-dependent SEMF coefficients. This leads to two major, observable consequences: a "Dynamic Valley of Stability" and, most critically, "Geometrically-Induced Fission" (GIF). The GIF mechanism—whereby extreme curvature drives the fissility parameter $x(\mathcal{K}) \geq 1$—provides a new, gravitationally-driven channel for nuclear decay that is absent from all standard models. In summary, this paper finds GIMV to be a coherent, well-motivated, and observationally accessible framework. It moves beyond standard physics by linking mass to geometry, and it offers concrete predictions that can be rigorously tested with the coming generation of multimessenger astrophysical data.

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