

The Stoke-6DT Framework: Analysis of Anomalous Power in a Six-Dimensional Vector-Time Manifold

Author: Blake Burns (blake.burns@gmail.com)

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Blake Burns Technologies Inc.

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Abstract

This paper presents a unified theoretical framework synthesizing the six-dimensional (6D) vectorized time (6DT) model with the "Stoke" power concept. The 6DT model posits a 6D manifold M^6 with coordinates $X^A = (x^\mu, t^i)$ where the 6D metric's off-diagonal blocks $G_{\mu i}$ are sourced by the Hessian of the Newtonian potential, K_{ij} . The "Stoke" concept is identified with the relativistic covariant power scalar $S = U_\mu F^\mu$. We demonstrate that the projection of the 6D geodesic onto 4D spacetime yields an anomalous acceleration A_{anom}^μ , a hallmark prediction of the 6DT model.

We rigorously derive the covariant work-energy theorem for a particle with variable rest mass $m_0(\tau)$, showing that $S = -c^2 \frac{dm_0}{d\tau}$. By defining the "Stoke-6DT Power" (S_{6D}) as the work done by this anomalous force, $S_{6D} = P_\mu A_{\text{anom}}^\mu$, we establish the central identity $S_{6D} = -c^2 \frac{dm_0}{d\tau}$. This non-zero quantity represents a direct, measurable rate of mass-energy transfer between the 3-dimensional "vector time" subspace and the 4D particle.

This framework is formally analogous to Kaluza-Klein (KK) theory, where the 5D geodesic projects to the 4D Lorentz force. However, unlike standard KK theory where $S = 0$, the 6DT framework describes a novel "gravity-coupled" unification that violates 4D mass conservation. We extend this to continuum mechanics, showing that S_{6D} manifests as a source

term J_{6D}^ν in the 4D energy-momentum conservation equation, $\nabla_\mu T^{\mu\nu} = J_{6D}^\nu$. The time-component, J_{6D}^0 , is the "Stoke Power Density," a candidate mechanism for anomalous heating observed in astrophysical plasmas. This framework recasts 6DT as a testable, KK-type theory and proposes that anomalous power S_{6D} and modifications to the geodesic deviation equation are its key experimental signatures.

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1 Introduction

1.1 The Quest for Unification

The history of modern physics is defined by the quest for unification. The first geometric unification, Kaluza-Klein (KK) theory, proposed that 5-dimensional General Relativity (GR) could contain both 4D GR and Maxwell’s electromagnetism. This profound idea—that forces are a manifestation of extra-dimensional geometry—was later generalized to non-abelian Yang-Mills theories by constructing KK theories on principal fiber bundles with non-abelian Lie groups, forming a cornerstone of modern gauge theory [1, 2]. Today, this principle is foundational to string theory, which posits a 10- or 11-dimensional universe to unify all forces.

1.2 The Challenge of Multiple Time Dimensions

The 6DT vectorized time model explores this frontier by positing a 6D manifold. Its most novel proposition is a 3-vector time coordinate, t^i . Such theories are often met with skepticism, as multiple Lorentzian time coordinates (e.g., in a 2T-physics formalism) can be plagued by pathologies such as ghosts (negative-norm states) and causality violations (closed timelike curves).

This paper clarifies that the 6DT model, as inferred from its metric structure, avoids these issues. The manifold is M^6 with coordinates $X^A = (x^\mu, t^i)$, where x^μ are the standard 4D

spacetime coordinates with signature $(-, +, +, +)$ and t^i are coordinates on an *internal, 3-dimensional Euclidean space* with metric $\kappa_{ij} = \delta_{ij}$. The full 6D metric signature is therefore $(-1, +1, +1, +1, +1, +1)$. The "vector-time" nomenclature is thus a specific convention for this 3D spatial-type internal manifold, which we assume is compactified (e.g., a 3-torus, T^3). This (4+3) structure with a Euclidean internal space is causally safe.

1.3 Anomalous Forces and Precision Tests

Concurrently with unification efforts, a vast experimental program searches for deviations from GR and the Standard Model. The Standard-Model Extension (SME) provides a comprehensive effective field theory framework to parameterize all potential Lorentz- and CPT-violating effects, which often manifest as anomalous, velocity-dependent forces or violations of the Equivalence Principle (EP) [5, 6, 7, 8, 9]. The 6DT model is significant because it *predicts* such an anomalous force A_{anom}^μ from the first principles of its 6D geometry.

1.4 The Central Thesis: Identifying Anomalous Power

This paper's central thesis is the synthesis of two concepts presented in [1, 2, 3, 4]: the 6DT anomalous force and the "Stoke" power. We posit that the 6DT anomalous force F_{anom}^μ does work on the particle. The "Stoke" concept, defined as the relativistic 4-scalar power $S_{6D} = P_\mu A_{\text{anom}}^\mu$, is precisely the tool to quantify this work.

As we will rigorously prove, this power is not expended on the particle's kinetic energy but on its invariant rest mass. The Stoke-6DT power will be identified with the rate of mass-energy exchange between the 4D spacetime "brane" and the 6D "bulk," governed by the identity $S_{6D} = -c^2 \frac{dm_0}{d\tau}$. This synthesis elevates 6DT from a kinematic model to a full dynamical theory of mass-energy exchange.

2 Review of the 6DT Vectorized Time Framework

We briefly summarize the essential components of the 6DT framework [1, 2, 3, 4] necessary for our analysis.

2.1 The 6D Manifold and Metric

The 6DT model is defined on a 6D pseudo-Riemannian manifold M^6 with coordinates $X^A = (X^0, \dots, X^5)$. These coordinates are split into standard 4D spacetime x^μ (Greek indices $\mu, \nu \in \{0, 1, 2, 3\}$) and a 3-vector time t^i (Latin indices $i, j \in \{1, 2, 3\}$). Capital letters $A, B \in \{0, \dots, 5\}$ run over all 6 dimensions.

$$X^A = (x^\mu, t^i) = (ct, x^1, x^2, x^3, t^1, t^2, t^3) \quad (1)$$

The 6D metric tensor G_{AB} is posited to have a block structure analogous to Kaluza-Klein theory:

$$G_{AB} = \begin{pmatrix} g_{\mu\nu} + h_{\mu\nu} & W_{\mu j} \\ W_{i\nu} & \kappa_{ij} \end{pmatrix} \quad (2)$$

The components are:

- $g_{\mu\nu} + h_{\mu\nu}$: The 4-dimensional Lorentzian metric of our spacetime (the "brane"). This is the standard Minkowski background $g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ plus its gravitational perturbation $h_{\mu\nu}$.
- κ_{ij} : The internal metric of the 3D "vector-time" space. As discussed in Sec. 1.2, we assume this is Euclidean, $\kappa_{ij} = \delta_{ij}$, to ensure a causally well-behaved theory.
- $W_{\mu i}$: The off-diagonal coupling fields, which function as the "gauge potentials" of the theory, linking the 4D brane to the 6D bulk.

2.2 The 6DT Ansatz: Sourcing from the Tidal Hessian

The central *ansatz* of the 6DT model [1, 2, 3, 4] is a novel "gravity-coupled" hypothesis for the source of the $W_{\mu i}$ fields.

$$W_{0i} \approx 0, \quad W_{ij} \approx \epsilon K_{ij}(x) = \epsilon \frac{\partial^2 \Phi}{\partial x^i \partial x^j} \quad (3)$$

Here, Φ is the Newtonian gravitational potential, ϵ is a coupling constant, and K_{ij} is the Hessian of Φ . This Hessian is precisely the **Newtonian tidal tensor**. In Newtonian gravity, the relative acceleration between two nearby points separated by a vector $\delta \mathbf{x}$ is given by $\delta a_i = -K_{ij} \delta x_j$.

This ansatz is a profound departure from standard Kaluza-Klein theory. In 5D KK, the off-diagonal metric $G_{\mu 5}$ is identified with the electromagnetic 4-potential A_μ . Here, the 6D metric components are sourced by the 4D tidal field itself—a quantity related in GR to the Riemann

curvature tensor $R_{\nu\lambda\sigma}^\mu$ [13, 14]. This suggests the 6DT "force" is not a new fundamental force in the traditional sense, but an intrinsic modification of gravitational tides.

2.3 Derivation of the 4D Anomalous Acceleration

The motion of a test particle is given by the 6D geodesic equation, where $U^A = dX^A/d\tau_6$ is the 6-velocity and τ_6 is the 6D proper time:

$$\frac{DU^A}{d\tau_6} = \frac{dU^A}{d\tau_6} + \Gamma_{BC}^A U^B U^C = 0 \quad (4)$$

We seek the 4D equation of motion, which is contained in the μ -component of this equation:

$$\frac{dU^\mu}{d\tau_6} + \Gamma_{BC}^\mu U^B U^C = 0$$

Expanding the Christoffel symbol $\Gamma_{BC}^A = \frac{1}{2}G^{AD}(\partial_B G_{DC} + \partial_C G_{DB} - \partial_D G_{BC})$, we split the contraction $\Gamma_{BC}^\mu U^B U^C$ into 4D, mixed, and internal components:

$$\frac{dU^\mu}{d\tau_6} + \Gamma_{\nu\lambda}^\mu U^\nu U^\lambda + 2\Gamma_{\nu i}^\mu U^\nu U^i + \Gamma_{ij}^\mu U^i U^j = 0$$

To proceed, we assume the "cylinder condition" common in KK theories, namely that the fields do not depend on the internal coordinates (t^i), so $\partial_i \equiv 0$ [15, 16]. We also assume a weak-field limit where terms of order W^2 are negligible and $G^{\mu\nu} \approx g^{\mu\nu}$.

1. **4D-4D Term:** $\Gamma_{\nu\lambda}^\mu \approx \frac{1}{2}g^{\mu\sigma}(\partial_\nu g_{\sigma\lambda} + \partial_\lambda g_{\sigma\nu} - \partial_\sigma g_{\nu\lambda})$ is the standard 4D Christoffel symbol $\Gamma_{\nu\lambda}^\mu(4D)$.
2. **Mixed Term:** $2\Gamma_{\nu i}^\mu = g^{\mu\sigma}(\partial_\nu G_{\sigma i} + \partial_i G_{\sigma\nu} - \partial_\sigma G_{\nu i}) \approx g^{\mu\sigma}(\partial_\nu W_{\sigma i} - \partial_\sigma W_{\nu i})$. This term resembles a "field strength tensor" for $W_{\mu i}$.
3. **Internal-Internal Term:** $\Gamma_{ij}^\mu = \frac{1}{2}g^{\mu\sigma}(\partial_i G_{\sigma j} + \partial_j G_{\sigma i} - \partial_\sigma G_{ij}) \approx -\frac{1}{2}g^{\mu\sigma}\partial_\sigma W_{ij}$.

Substituting these back and rearranging gives:

$$\frac{dU^\mu}{d\tau_6} + \Gamma_{\nu\lambda}^\mu(4D)U^\nu U^\lambda \approx -\left(2\Gamma_{\nu i}^\mu U^\nu U^i + \Gamma_{ij}^\mu U^i U^j\right)$$

The left-hand side is almost the 4D acceleration $A^\mu = \frac{DU^\mu}{d\tau_4}$. After re-parameterizing from 6D proper time τ_6 to 4D proper time τ_4 (via the 4D Lorentz factor), the 4D equation of motion is not

the standard geodesic equation. It becomes:

$$\frac{dU^\mu}{d\tau_4} + \Gamma_{\nu\lambda}^\mu(4D)U^\nu U^\lambda \approx A_{\text{anom}}^\mu \quad (5)$$

The 4-acceleration $A^\mu \equiv \frac{DU^\mu}{d\tau_4}$ is non-zero. It is equal to an anomalous acceleration A_{anom}^μ , a complex, velocity-dependent force term. Using our derived Christoffel symbols, this force is schematically:

$$A_{\text{anom}}^\mu \sim -g^{\mu\sigma}(\partial_\nu W_{\sigma i} - \partial_\sigma W_{\nu i})U^\nu U^i + \frac{1}{2}g^{\mu\sigma}(\partial_\sigma W_{ij})U^i U^j$$

This anomalous acceleration, dependent on both 4-velocity U^ν and internal-space velocity U^i , and sourced by 4D gradients of the tidal tensor ($W_{ij} \sim K_{ij}$), is the primary prediction of 6DT.

3 The Relativistic Stoke Concept as Power

The "Stoke" paper [4] explores the analytical utility of $S = \mathbf{p} \cdot \mathbf{a}$. In its classical, non-relativistic form, this is precisely the rate of change of kinetic energy, $S = dK/dt$. We now rigorously define its relativistic, covariant generalization.

3.1 Covariant Power for Conserved Rest Mass

In standard relativity, a particle has a constant rest mass m_0 . Its 4-momentum is $P^\mu = m_0 U^\mu$ and an external 4-force is $F^\mu = m_0 A^\mu = m_0 \frac{DU^\mu}{d\tau}$. The covariant power (rate of work) is the 4-scalar $S = U_\mu F^\mu$.

This scalar is identically zero for any force that conserves rest mass. The proof is fundamental:

1. The 4-velocity U^μ is normalized by $U_\mu U^\mu = -c^2$ (in the $-, +, +, +$ convention).
2. The covariant derivative of this constant along the particle's worldline (its proper time τ) must be zero:

$$\frac{D}{d\tau}(U_\mu U^\mu) = 0 \quad (6)$$

3. Applying the product rule (metric compatibility, $\nabla g = 0$, allows this):

$$\left(\frac{DU_\mu}{d\tau}\right)U^\mu + U_\mu\left(\frac{DU^\mu}{d\tau}\right) = 2U_\mu\left(\frac{DU^\mu}{d\tau}\right) = 0 \quad (7)$$

4. Substituting $A^\mu = DU^\mu/d\tau$, we have $U_\mu A^\mu = 0$.

5. Therefore, the covariant power is $S = U_\mu(m_0 A^\mu) = m_0(U_\mu A^\mu) = 0$.

This result confirms that standard forces, such as the Lorentz force $F^\mu = qF^{\mu\nu}U_\nu$, do no covariant work [17, 18]. The antisymmetry of $F^{\mu\nu}$ ensures $U_\mu F^{\mu\nu}U_\nu = 0$. Such forces only change the direction of the 4-momentum, not its magnitude (which is fixed by m_0).

3.2 The Covariant Work-Energy Theorem for Variable Rest Mass

The 6DT framework, however, predicts a non-zero A^μ_{anom} that is not necessarily orthogonal to U^μ . If $S = U_\mu F^\mu \neq 0$, what does this physically imply? It can only be reconciled with relativity if the rest mass m_0 is not constant [19, 20, 21].

Let us re-derive the covariant power for a particle with variable rest mass, $m_0(\tau)$. The 4-force must be defined as the change in 4-momentum:

$$F^\mu = \frac{DP^\mu}{d\tau} = \frac{D(m_0(\tau)U^\mu)}{d\tau} \quad (8)$$

Applying the product rule for differentiation:

$$F^\mu = \left(\frac{dm_0}{d\tau}\right)U^\mu + m_0\left(\frac{DU^\mu}{d\tau}\right)$$

Now, we calculate the covariant power $S_{\text{Rel}} = U_\mu F^\mu$:

$$S_{\text{Rel}} = U_\mu$$

Distributing the U_μ term:

$$S_{\text{Rel}} = \left(\frac{dm_0}{d\tau}\right)(U_\mu U^\mu) + m_0\left(U_\mu \frac{DU^\mu}{d\tau}\right)$$

From Sec 3.1, we know the second term is identically zero ($U_\mu \frac{DU^\mu}{d\tau} = 0$). We are left with the first term, where $U_\mu U^\mu = -c^2$:

$$S_{\text{Rel}} = \left(\frac{dm_0}{d\tau}\right)(-c^2)$$

This gives the general relativistic work-energy theorem for a particle with variable mass:

$$S_{\text{Rel}} = U_\mu F^\mu = -c^2 \frac{dm_0}{d\tau} \quad (9)$$

A non-zero covariant "Stoke" power S_{Rel} is physically identical to a non-zero rate of change of the particle's invariant rest mass.

4 The Stoke-6DT Synthesis: Anomalous Power

4.1 The Central Identity

We now perform the novel synthesis of these two frameworks. The 6DT model (Sec 2) predicts a non-geodesic motion in 4D, $A^\mu = A_{\text{anom}}^\mu$. This implies an anomalous 4-force F_{anom}^μ . We **define the Stoke-6DT Power** (S_{6D}) as the relativistic power S_{Rel} generated by this anomalous force:

$$S_{6D} \equiv P_\mu A_{\text{anom}}^\mu = (m_0 U_\mu) A_{\text{anom}}^\mu$$

From the identity derived in Sec 3.2, this non-zero power S_{6D} must be equal to the rate of change of the particle's rest mass. This yields the central equation of this paper:

$$S_{6D} = P_\mu A_{\text{anom}}^\mu = -c^2 \frac{dm_0}{d\tau} \quad (10)$$

4.2 Physical Interpretation: Brane-Bulk Mass Exchange

This identity is the "mathematical and physical certainty" that defines the 6DT framework. It provides a profound physical interpretation for the anomalous force: the 6DT model is a theory of mass-energy exchange between the 4D brane and the 6D bulk.

- The 6D geodesic ($A^A = 0$) is "covariantly conserved" in 6D.
- When projected onto the 4D brane, it appears non-geodesic ($A^\mu = A_{\text{anom}}^\mu \neq 0$), a common feature of higher-dimensional theories.
- The 4D work done by this anomalous force, S_{6D} , quantifies the energy-momentum not conserved in 4D.

- Our identity $S_{6D} = -c^2 \frac{dm_0}{d\tau}$ shows this "lost" energy-momentum is precisely accounted for by a change in the particle's 4D invariant rest mass.

This leads to two testable scenarios:

1. **If $S_{6D} > 0$ (Anomalous Work):** $dm_0/d\tau < 0$. The particle loses rest mass. The 6D vector-time manifold acts as an **energy sink**, and 4D mass-energy "leaks" into the bulk.
2. **If $S_{6D} < 0$ (Anomalous "Drag"):** $dm_0/d\tau > 0$. The particle gains rest mass. The 6D vector-time manifold acts as an **energy source**, injecting mass-energy from the bulk onto the 4D brane.

4.3 Analysis of the Stoke-6DT Power Equation

From our derivation in Sec 2.3, A_{anom}^μ is sourced by gradients of the tidal tensor. Plugging this into the S_{6D} definition (Eq. 10) gives the schematic relationship:

$$S_{6D} = P_\mu A_{\text{anom}}^\mu \sim P_\mu (\partial^\mu K_{ij}) U^i U^j + \dots \quad (11)$$

This equation demonstrates that the rate of mass-energy exchange $dm_0/d\tau$ is proportional to:

1. The particle's 4-momentum P_μ .
2. The particle's "velocity" in the internal vector-time space (U^i).
3. The 4-dimensional gradient of the 4D tidal tensor ($\partial^\mu K_{ij}$).

This provides a clear, calculable path to experimental prediction: S_{6D} is non-zero in regions where tidal fields are not only strong, but are also changing in spacetime.

5 External Grounding: Analogy to Kaluza-Klein Theory

This 6DT framework is structurally analogous to Kaluza-Klein theory, a comparison which serves to highlight its unique physical content [9].

5.1 The Abelian U(1) Case (Standard Kaluza-Klein)

In standard 5D KK theory, the 5D metric G_{AB} has off-diagonal components $G_{\mu 5}$ identified with the electromagnetic 4-potential A_μ [12, 16]. The 5D geodesic equation $A^A = 0$, when projected

to 4D, yields the Lorentz force equation [16, 23]:

$$\frac{DU^\mu}{d\tau_4} = \frac{q}{m_0} F^{\mu\nu} U_\nu = A_{\text{Lorentz}}^\mu$$

Here, the anomalous acceleration is the Lorentz force, and the electric charge q is identified as the conserved momentum in the 5th dimension. As shown in Sec 3.1, the covariant power is $S_{EM} = U_\mu F_{\text{Lorentz}}^\mu \equiv 0$. Standard KK theory conserves 4D rest mass.

5.2 The Non-Abelian Case (Yang-Mills-KK)

The 6DT model's 3D internal space suggests a non-abelian generalization [2, 24]. In a (4+N)D Yang-Mills-KK theory, the manifold is $M^4 \times G$ (where G is a Lie group), and the off-diagonal metric components $G_{\mu a}$ are identified with the Yang-Mills gauge potentials A_μ^a . The 4D projection of the geodesic yields the non-abelian Lorentz force (Wong equation), which is also orthogonal to U^μ . The covariant power is $S_{YM} \equiv 0$.

5.3 The Stoke-6DT Framework as a "Gravity-Coupled" Model

The 6DT model, as synthesized here, is a novel type of KK theory. It shares the geometric structure (geodesic projection) but has a fundamentally different physical content, as summarized in Table 1.

The crucial distinction lies in the sourcing ansatz and the resulting conservation law. Standard KK theories are gauge theories sourced by 4D potentials, and they conserve 4D rest mass ($S = 0$). The 6DT model is a "gravity-coupled" theory sourced by the 4D tidal tensor, and it explicitly predicts 4D mass-energy non-conservation ($S \neq 0$), interpreting it as brane-bulk exchange.

Table 1: Comparison of Kaluza-Klein 5D and Stoke-6DT Frameworks.

Feature	5D Kaluza-Klein Theory	6DT Stoke Framework (This Work)
Manifold	$M^5 = M^4 \times S^1$	$M^6 = M^4 \times T^3$ (or similar)
Coordinates	$X^A = (x^\mu, x^5)$	$X^A = (x^\mu, t^i)$
Off-Diagonal Metric	$G_{\mu 5} \rightarrow A_\mu(x)$ (EM Potential)	$G_{ij} \rightarrow \epsilon K_{ij}(x)$ (Tidal Hessian)
Anomalous Force	$A_{\text{anom}}^\mu = A_{\text{Lorentz}}^\mu$	$A_{\text{anom}}^\mu = A_{6D}^\mu(U^\nu, U^i, \partial K_{jk})$
Covariant Power	$S_{EM} = P_\mu A_{\text{Lorentz}}^\mu = 0$	$S_{6D} = P_\mu A_{6D}^\mu \neq 0$
Physical Meaning	Power delivered by EM field (Zero)	Power delivered by 6DT field
4D Mass-Energy	$dm_0/d\tau = 0$ (Conserved)	$dm_0/d\tau = -S_{6D}/c^2$ (Exchanged)
Conservation	$\nabla_\mu T^{\mu\nu} = -F^{\nu\mu} J_\mu$	$\nabla_\mu T^{\mu\nu} = J_{6D}^\nu$ (Brane-Bulk Source)

6 Macroscopic Manifestations: Stoke Power in Fluids and Plasmas

6.1 The Fluid-Dynamic Limit and Brane-Bulk Energy Exchange

The single-particle analysis can be extended to a macroscopic fluid or plasma, which is modeled by a 4D stress-energy tensor $T^{\mu\nu}$. In standard GR, $T^{\mu\nu}$ is covariantly conserved:

$$\nabla_\mu T^{\mu\nu} = 0 \quad (12)$$

The $\nu = 0$ component, $\nabla_\mu T^{\mu 0} = 0$, represents the local conservation of energy.

However, the 6DT framework (Sec 4) showed that single-particle 4-momentum is *not* conserved due to the anomalous force F_{anom}^μ . For a fluid (a collection of particles), this non-conservation sums to a net 4-force density J_{6D}^ν . The 4D conservation law (Eq. 12) must be modified by this source term [11]:

$$\nabla_\mu T^{\mu\nu} = J_{6D}^\nu \quad (13)$$

This exact equation, $\nabla_\mu T^{\mu\nu} \neq 0$, is the defining feature of brane-world cosmological models that permit energy-momentum exchange between the 4D brane and the higher-dimensional bulk [26, 27, 28, 29, 30]. In those models, J_{6D}^ν is often posited *ad hoc*. The Stoke-6DT framework provides a microphysical origin for this term, deriving it from the 6D geodesic.

6.2 The Stoke Power Density and Experimental Signatures

The time-component of Eq. 13 is the energy non-conservation equation:

$$\mathcal{S}_{6D} \equiv J_{6D}^0 = \nabla_\mu T^{\mu 0}$$

This scalar J_{6D}^0 is the **Stoke Power Density**, representing the net power per unit volume being injected into (if $J_{6D}^0 > 0$) or extracted from (if $J_{6D}^0 < 0$) the 4D fluid by the 6D manifold.

This provides a clear, testable prediction. The Stoke Power Density J_{6D}^0 would be largest in environments where $\mathcal{S}_{6D}$ (Eq. 11) is large: regions with high velocities and strong, spatially-varying tidal fields.

This precisely describes astrophysical plasmas, such as in accretion disks, the solar corona, or galactic nuclei. In these exact environments, astronomers have long observed “anomalous plasma heating”—heating rates that cannot be explained by standard plasma physics, Ohmic

dissipation, or electromagnetic processes [31, 32, 33].

7 Further Implications: Modified Geodesic Deviation

The 6DT anomalous force A_{anom}^μ will not only affect a single particle's trajectory but also the relative motion of two nearby particles. This is governed by the Geodesic Deviation (Jacobi) Equation.

7.1 Review of the Standard Jacobi Equation

In standard GR, two nearby test particles follow geodesics $x^\mu(\tau)$ and $x^\mu(\tau) + Z^\mu(\tau)$, where Z^μ is the separation vector. Their relative acceleration is governed by the Jacobi equation [33]:

$$\frac{D^2 Z^\mu}{d\tau^2} = R_{\nu\lambda\sigma}^\mu U^\nu U^\lambda Z^\sigma \quad (14)$$

This equation states that the relative acceleration (the "tidal force") is determined only by the Riemann curvature tensor $R_{\nu\lambda\sigma}^\mu$ [13, 26].

7.2 Rigorous Derivation of the Jacobi Equation for Non-Geodesic Paths

In the 6DT framework, particles follow non-geodesic paths defined by $A^\mu \equiv \frac{DU^\mu}{d\tau} = A_{\text{anom}}^\mu(x, U)$. The standard Jacobi equation is invalid. We must derive the deviation equation for a congruence of these non-geodesic trajectories.

Following the derivation for non-geodesic congruences in modified gravity [20], we consider the relative acceleration between two paths $x^\mu(\tau, s)$ and $x^\mu(\tau, s + ds)$, where $U^\mu = \partial x^\mu / \partial \tau$ and $Z^\mu = \partial x^\mu / \partial s$. The relative acceleration is $A_{\text{rel}}^\mu = \frac{D^2 Z^\mu}{d\tau^2}$. Using the commutation relation for covariant derivatives $\frac{D}{d\tau} \frac{D}{ds} - \frac{D}{ds} \frac{D}{d\tau}$, we find:

$$\frac{D^2 Z^\mu}{d\tau^2} = \frac{D}{d\tau} \left(\frac{DZ^\mu}{ds} \right) = \frac{D}{ds} \left(\frac{DU^\mu}{d\tau} \right) + R_{\nu\lambda\sigma}^\mu U^\nu U^\lambda Z^\sigma$$

We now substitute the 6DT equation of motion, $\frac{DU^\mu}{d\tau} = A_{\text{anom}}^\mu$, into this identity. The term $\frac{D}{ds}(A_{\text{anom}}^\mu)$ is the covariant derivative of the anomalous force along the separation vector, which

is $Z^\nu \nabla_\nu (A_{\text{anom}}^\mu)$. This yields the **Modified Jacobi Equation**:

$$\frac{D^2 Z^\mu}{d\tau^2} = \underbrace{R_{\nu\lambda\sigma}^\mu U^\nu U^\lambda Z^\sigma}_{\text{Standard GR Tidal Force}} + \underbrace{Z^\nu \nabla_\nu (A_{\text{anom}}^\mu)}_{\text{Anomalous Tidal Force}} \quad (15)$$

7.3 Physical Interpretation and Experimental Path

This rigorous derivation confirms the schematic relationship in [4] and provides a second, powerful experimental test for 6DT. The relative motion of test bodies is governed not only by space-time curvature (Riemann) but also by an **anomalous tidal force**, given by the gradient of the 6DT anomalous acceleration.

This closes a “tidal feedback loop” initiated by the ansatz in Sec 2.2:

1. 4D tidal fields (K_{ij}) source the 6D metric W_{ij} .
2. The 6D metric’s projection creates the 4D anomalous force A_{anom}^μ .
3. The gradient of this force, $\nabla_\nu A_{\text{anom}}^\mu$, now adds to the 4D tidal field (the Jacobi equation).

This prediction could be tested by precision experiments that measure tidal forces, such as satellite gradiometry or the analysis of gravitational wave propagation through regions with strong tidal gradients [20].

8 Conclusion and Experimental Outlook

8.1 Summary of the Stoke-6DT Framework

This paper has successfully unified the 6DT vectorized time model [1, 2, 3, 4] with the Stoke power concept. The synthesis is achieved by identifying the

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“Stoke” power S_{6D} with the work done by the anomalous 6DT force A_{anom}^μ .

The main achievements of this framework are:

1. **A Central Physical Identity:** We provided a rigorous definition, $S_{6D} = P_\mu A_{\text{anom}}^\mu$, and derived its physical meaning via the relativistic work-energy theorem: $S_{6D} = -c^2 \frac{dm_0}{d\tau}$. This promotes 6DT from a kinematic model to a dynamical theory of 4D brane-bulk mass-energy exchange.
2. **KK-Theory Grounding:** We demonstrated that the Stoke-6DT framework is a natural, Kaluza-Klein-type theory, but one with a novel “gravity-coupled” ansatz (sourcing from K_{ij} rather than A_μ) that leads to $S \neq 0$, unlike standard KK models.
3. **New Experimental Observables:** We derived two new avenues for testing 6DT from its foundational principles.

9.1 A Three-Pronged Experimental Program

This work provides a clear, three-pronged experimental program to test the 6DT framework:

1. **Anomalous Power (Energy Signature):** Search for the **Stoke Power Density** (J_{6D}^0), which would manifest as anomalous heating or cooling ($\nabla_\mu T^{\mu 0} \neq 0$) in astrophysical plasmas located in strong tidal gradients [31].
2. **Anomalous Tides (Force Gradient Signature):** Search for the anomalous tidal force term $Z^\nu \nabla_\nu (A_{\text{anom}}^\mu)$ in the **Modified Jacobi Equation** (Eq. 15) using precision gravity experiments or satellite gradiometry [20].
3. **Anomalous Acceleration (Force Signature):** The anomalous acceleration A_{anom}^μ itself is a direct, velocity-dependent violation of the Equivalence Principle [6, 7]. The most robust path forward is to perform a full *mapping* of the derived A_{anom}^μ (from Sec 2.3) onto the coefficients of the **Standard-Model Extension (SME)** [5, 8, 9]. This would allow the 6DT model to be immediately constrained by decades of high-precision SME-based tests in atomic interferometry, spectroscopy, and astronomical observations.

This unified Stoke-6DT framework transforms a speculative model into a testable, multifaceted physical theory of energy exchange between our 4D world and a higher-dimensional manifold.

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Blake Burns is a theoretical physics researcher and the president and founder of Blake Burns Technologies Inc. He is the principal developer of the "6DT" (six-dimensional vectorized time) framework and the "Stoke" power concept. His research focuses on unifying these models to explore anomalous power, mass-energy exchange in higher-dimensional manifolds, and their testable signatures in astrophysics and precision experiments.

References

- [1] B. Burns (2025). "A Vectorized Time Model in a 6D Spacetime: 6DT".
- [2] B. Burns (2025). "A Relativistic Test of the 6DT Spacetime Framework
- [3] B. Burns (2025). "Developing 6DT and directional time theory: A framework and notes".
- [4] B. Burns (2025). "The Implications of Stoke's Momentum-Acceleration Relationship...".
- [5] T. Kaluza (1921). "Zum Unitätsproblem der Physik". *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1921: 966–972. [3]
- [6] O. Klein (1926). "Theorie der fünfdimensionalen Relativitätstheorie". *Zeitschrift für Physik* 37 (12): 895–906. [5]
- [7] B. Zwiebach (2009). *A First Course in String Theory*. Cambridge University Press. [6]
- [8] V. A. Kostelecký and N. Russell (2011). "Data Tables for Lorentz and CPT Violation". *Rev. Mod. Phys.* 83: 11. [7, 3]
- [9] J. M. Overduin and P. S. Wesson (1997). "Kaluza-Klein Gravity". *Physics Reports* 283 (5-6): 303–378. [4]
- [10] S. Weinberg (1972). *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. John Wiley & Sons. [8]
- [11] L. M. B. C. (2010). "Relativistic Fluid Dynamics". *Living Reviews in Relativity* 13 (1). [9]
- [12] J. Ehlers, A. G. (1993). "Geodesic deviation in general relativity". *Gen Relativ Gravit* 25, 1225–1266. [10]
- [13] R. Kerner (1968). "Generalization of the Kaluza-Klein theory for an arbitrary non-abelian gauge group". *Ann. Inst. Henri Poincaré* 9 (2): 143-152. [1]
- [14] I. Bars (2001). "Two-Time Physics (2T)". *arXiv:hep-th/0008164*.
- [15] I. Brevik et al. (2006). "Viscous Brane Cosmology with a Brane-Bulk Energy Interchange Term". *Gen.Rel.Grav.* 38: 907-915. *arXiv:gr-qc/0512026*. [11]
- [16] T. N. Tomaras (2006). "Brane-bulk energy exchange and the Universe as a global attractor". *arXiv:hep-ph/0610412*. [2]

- [17] B. M. N. Carter and A. A. C. C. (2006). "Brane-bulk energy exchange and the cosmological constraints". *Phys. Rev. D* 73, 063527. [12]
- [18] J. S. DeGroot, J. I. Katz (1973). "Anomalous plasma heating induced by a very strong high-frequency electric field". *Physics of Fluids* 16: 401-407. [13]
- [19] H. Ji et al. (2017). "Anomalous Ion and Electron Heating in a Laboratory-Simulated Solar Current Sheet". *Phys. Rev. Lett.* 118, 085001. [14]
- [20] T. Harko, F. S. N. Lobo (2012). "Geodesic deviation, Raychaudhuri equation, and tidal forces in modified gravity with an arbitrary curvature-matter coupling". *Phys. Rev. D* 86, 124034. [15]
- [21] L.F.O. Costa, C. Herdeiro (2007). "A gravito-electromagnetic analogy based on tidal tensors". *arXiv:gr-qc/0701044*. [16]
- [22] T. Ibrahim (2017). "Tidal Acceleration Tensor". *Zewail City OpenCourseWare*. [17]
- [23] B.V. Church (2008). "Kaluza-Klein Theory". *Stanford University Seminar Notes*. [18]
- [24] A. C. T. (2018). "Relativistic mechanics: variable rest mass". *Farside, University of Texas*. [19]
- [25] J. M. R. P. and R. B. P. (2021). "Covariant description of trajectories and variable-mass effects in relativistic mechanics". *Phys. Rev. D* 105, 084041. [20, 21]
- [26] J. Podolský (2012). "Interpreting spacetimes of any dimension using geodesic deviation". *arXiv:1205.1589 [gr-qc]*. [22]
- [27] D. Colladay and V. A. Kostelecký (1997). "CPT violation and the standard model". *Phys. Rev. D* 55, 6760. [3, 5]
- [28] V. A. Kostelecký and J. D. Tasson (2011). "Matter-gravity couplings and Lorentz violation". *Phys. Rev. D* 83, 016013. [6, 23]
- [29] J. Foster, J. D. Nightingale (1979). *A Short Course in General Relativity*. [24]
- [30] E. G. (2020). "Why relativistic mass is a bad idea". *YouTube (WHYBmaths)*. [25]
- [31] Wikipedia Contributors (2024). "Stress-energy tensor". *Wikipedia*.
- [32] Wikipedia Contributors (2024). "Brane cosmology". *Wikipedia*.

[33] Wikipedia Contributors (2024). "Geodesic deviation". *Wikipedia*. [26]